SOLUTIONS

Solution 1 :

1°/ On a : $x^2 + 3x - 4 = (x-1)(x+4)$

$$\lim_{x\to 1}\frac{x^2+3x-4}{x-1} = \lim_{x\to 1}\frac{(x-1)(x+4)}{(x-1)} = \lim_{x\to 1}(x+4) = 5$$

2°/ On pose $X = x - 1$ lorsque $x \to 1$ alors $x \to 0$

d'où $$\lim_{x\to 1}\frac{\cos(x-1)-1}{x^2+2x-3} = \lim_{x\to 0}\frac{\cos X - 1}{X(X+4)} = \lim_{x\to 0}\frac{-1}{X+4}\cdot\frac{1-\cos X}{X} = 0$$

3°/ $$\lim_{x\to+\infty} x^2\left(1-\cos\left(\frac{1}{x}\right)\right) = \lim_{x\to+\infty}\frac{1-\cos\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)^2} = \lim_{x\to 0^+}\frac{1-\cos X}{X^2} = \frac{1}{2}$$

4°/ On a : $E(x) \le x \le E(x)+1 \Rightarrow x-1 < E(x) \le x$

Pour $x > 0$ on a : $x + 1 > 0$ d'où

$$\frac{x-1}{x+1} < \frac{E(x)}{x+1} \le \frac{x}{x+1}$$

$$\lim_{x\to+\infty}\frac{x-1}{x+1} = 1 \text{ et } \lim_{x\to+\infty}\frac{x}{x+1} = 1 \qquad \text{D'où : } \lim_{x\to+\infty}\frac{E(x)}{x+1} = 1$$

Solution 2 :

$$1°/\lim_{x\to 1}\frac{\sqrt{x+3}-2}{x-1} = \lim_{x\to 1}\frac{(\sqrt{x+3}-2)(\sqrt{x+3}+2)}{(x-1)\cdot(\sqrt{x+3}+2)} = \lim_{x\to 1}\frac{(x+3)-4}{(x-1)(\sqrt{x+3}+2)}$$

$$= \lim_{x\to 1}\frac{x-1}{(x-1)\cdot(\sqrt{x+3}+2)} = \lim_{x\to 1}\frac{1}{\sqrt{x+3}+2} = \frac{1}{4}$$

$$2°/\lim_{x\to 1}\frac{\sqrt{x+2}-\sqrt{2x+1}}{x-1} = \lim_{x\to 1}\frac{(\sqrt{x+2}-\sqrt{2x+1})(\sqrt{x+2}+\sqrt{2x+1})}{(x-1)(\sqrt{x+2}+\sqrt{2x+1})}$$